



Study of conjugate heat transfer for a magnetohydrodynamic fluid in a semicircular cavity.

A thesis
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DECLARATION

We hereby declare that this thesis is our work and to the best of our knowledge it contains no materials previously published or written by another person, or have been accepted for the award of any other degree or diploma at Sonargaon University or any other educational institutions. We also declare that the intellectual content of this thesis is the product of our work and any contribution made to the research by others, with whom I have worked at Sonargaon University or elsewhere, is explicitly acknowledged.

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[Authors]

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ABSTRACT

This thesis investigates the heat transfer effects within a semicircular cavity under the influence of a uniform magnetic field, with a focus on magnetohydrodynamic (MHD) flows. Conducted within the framework of the Department of Mechanical Engineering at Sonargaon University, Dhaka, Bangladesh, the study addresses the complex interplay of fluid dynamics, heat transfer, and magnetic field interactions.

The research begins with the formulation of a comprehensive mathematical model, aligning with the academic standards of the Department. The model, describing the continuum of continuity, momentum, and energy equations, is appropriately nondimensionalized to enhance comparability and facilitate a clear interpretation of dimensionless parameters.

Numerical simulations are performed using the finite element method (FEM), consistent with the computational practices endorsed by the Department. Special attention is given to mesh generation, tailored to the computational resources available at Sonargaon University, to ensure high spatial resolution near the corrugated surface.

Visualization tools such as streamlines and isotherms are employed to effectively communicate fluid flow patterns and temperature distribution within the semicircular cavity, adhering to visualization standards set by the Department.

A comprehensive parametric study is conducted, varying Rayleigh number, and heating ratio, in line with numerical simulation practices. The analysis focuses on the effects of these parameters on streamlines, isotherms, velocities, and Nusselt number.

Results indicate that an increase in the Rayleigh number and heating ratio correlates with heightened heat transfer rates.

Validation against existing models is performed following the Department's standards for verification and comparison.

CHAPTER 1

Introduction

1.1 Overview

This thesis embarks on a comprehensive exploration of heat transfer phenomena within a semicircular cavity, influenced by the presence of a uniform magnetic field. The study addresses the intricate dynamics of natural convection in a magneto hydrodynamic (MHD) fluid filling the semicircular cavity, particularly under varying conditions of Rayleigh number, and height ratio. The analysis focuses on elucidating the impact of key parameters, such as the, Rayleigh number, height ratio, on the flow patterns, temperature distribution and Nusselt number throughout the entire cavity domain.

1.2 Objective

The primary objective of this thesis is to establish a holistic understanding of heat transfer effects within a semicircular cavity subjected to a uniform magnetic field. More key objectives are mention below:

Mathematical Modeling: We used a comprehensive mathematical model incorporating the governing partial differential equations, encompassing continuity, momentum, and energy equations.

Nondimensionalization: We nondimensionalized the system to facilitate a clearer interpretation of the dimensionless parameters and to align the study with established practices in fluid dynamics and heat transfer.

Numerical Simulation: We utilized the finite element method (FEM) to numerically solve the nondimensionalized equations.

Parametric Study: We conducted a systematic computational experiment by varying key parameters, including the Rayleigh number and height ratio. Analyzed and interpret the effects of these parameters on streamlines, isotherms and Nusselt number.

Visualization: Visualized the flow patterns and temperature distribution within the entire cavity domain using streamlines and isotherms. We interpreted and analyzed the visual representation to gain insights into the complex fluid dynamics and heat transfer behaviors.

Conclusions: We summarized the key findings, emphasizing the influence of the Rayleigh number and height ratio on Nusselt number.

This research contributes to the broader understanding of heat transfer phenomena in magnetohydrodynamic flows within complex geometries, offering insights with implications for various engineering applications.

1.3 Conjugate Heat Transfer

Conjugate heat transfer involves simultaneously analyzing heat transfer in interacting solid and fluid domains. It's crucial in scenarios where temperature distribution in one medium affects the other. The analysis requires solving heat transfer equations in both the solid and fluid regions.

1.4 Magneto hydrodynamic Fluid

Magnetohydrodynamic (MHD) fluid refers to a conductive fluid, such as plasma or liquid metal, that interacts with a magnetic field. The movement of the conductive fluid induces electric currents, leading to complex interactions between fluid dynamics and electromagnetic forces. This phenomenon is studied in astrophysics, geophysics, fusion research, and certain engineering applications.

1.5 Darcy Number

The Darcy number is a dimensionless quantity used in fluid mechanics to characterize the permeability of porous media. It is defined as the ratio of the permeability of a substance to its viscosity.

$$Da = \frac{k}{d^2}$$

where

K is the permeability of the medium.

D is the characteristic length of the particle.

1.6 Rayleigh number

Rayleigh Number: The Rayleigh number is defined as the ratio of buoyancy forces to viscous forces and is given by:

$$Da = \frac{g\beta\Delta TL^3}{\nu\alpha}$$

Here,

ν g is the acceleration due to gravity,

β is the thermal expansion coefficient,

ΔT is the temperature difference,

L is a characteristic length (related to the dimensions of the cavity),

ν is the kinematic viscosity, and $\nu \alpha$ is the thermal diffusivity.

1.7 Nusselt number

The Nusselt number is the ratio of conductive resistance to the convective resistance.

$$Nu = \frac{R_{\text{Conduction}}}{R_{\text{Convectio}}}$$

1.8 Hartmann Number

The Hartmann number (Ha) is the ratio of electromagnetic force to the viscous force.

$$Ha = BL \sqrt{\frac{\sigma}{\mu}}$$

where

B is the magnetic field intensity,

L is the characteristic length scale,

σ is the electrical conductivity,

μ is the dynamic viscosity.

1.9 Height Ratio

The height ratio of a semicircular cavity refers to the ratio of the height (vertical distance) of the cavity to the radius of the semicircular shape. In a semicircle, the height ratio is typically expressed as the height (h) divided by the radius (r). Mathematically, the height ratio (h/r) can be represented as:

$$\text{Height Ratio (h/r)} = \frac{h}{r}$$

1.10 Velocity Streamlines

The velocity streamlines in a semicircular cavity refers to the path traced by fluid particles within the cavity as they move with the flow. In a steady state situation, the streamline represents the direction and magnitude of the fluid velocity at different points within the cavity.

1.11 Isotherm in semicircular cavity

Isotherms in a semicircular cavity represent lines connecting points with the same temperature within the cavity. These lines provide a visual depiction of the temperature distribution. The specific pattern of isotherms depends on factors like the initial conditions, heat sources or sinks, and the thermal properties of the material.

CHAPTER 2

Literature Review

Natural convection, a heat transfer process where fluids move on their own, is not only a part of our daily lives but is also crucial for making things work better in industries. Unlike forced convection, where we actively move fluids, natural convection happens naturally because hot fluids rise (they're less dense) and cold fluids sink (they're more dense) due to gravity.

Understanding natural convection is vital for various applications in industries. It plays a key role in improving performance, enhancing aerodynamics, and addressing energy, heat transfer, and light-thermal conversion. For example, it helps in making energy-efficient windows with special glass panels. These windows can resist the outside temperature, keeping the inside comfortable and saving a lot of energy.

Moreover, natural convection has numerous engineering applications. It contributes to the cooling and heating of houses, the design of solar collectors, preventing electronic devices from overheating, creating micro-electromechanical systems (MEMS), and aiding in processes like food manufacturing and metallurgical industry. In essence, the study of natural convection is essential for making various technologies more efficient and effective. For detailed analysis on the applications the readers are referred to [1–3].

Unlike the simpler flow challenges encountered in open channels, dealing with the flow and heat transfer issues in enclosed cavities is more complex. Various obstacles, such as the shape of the cavity, the conditions at its boundaries, the specific governing equations, and the pursuit of the best numerical solution, add layers of difficulty. This research field is exceptionally significant due to its unavoidable applications, including heating and cooling systems for houses, microprocessors, air conditioning, and solar collectors.

Researchers have consistently aimed to enhance the performance of these processes, leading to a substantial body of literature on the subject. Both experimental and mathematical studies have been conducted by numerous researchers. This overview focuses on the mathematical aspects. For instance, a study employed a numerical approach, specifically the finite volume method, to analyze the buoyancy-driven flow and heat transfer of nanofluid within a two-dimensional rectangular enclosure [4]. Another investigation used the SIMPLE algorithm to numerically study the flow and heat transfer in a lid-driven cavity with differentially heated side walls [5]. The study provided a detailed analysis of how various parameters affect heat transfer rates, streamlines, and isotherms.

Inclined square cavities were also considered in research conducted by Abu-Nada and Oztop [6] to study natural convection heat transfer. The effects of different inclination

angles on flow and heat transfer characteristics were analyzed, employing the Finite Volume method to obtain numerical solutions.

Various cavity shapes are integral to engineering applications, including triangular [7, 8, 9], rectangular [10, 5, 11, 12], trapezoidal [13, 14, 15], and other uniquely designed cavities [16, 17, 18, 19]. A notable geometric variation involves cavities with corrugated walls, which find applications in solar collectors, house design, and more.

The article undertakes a thorough numerical exploration of natural convection featuring magnetohydrodynamic (MHD) fluid within a semicircular cavity. The content unfolds seamlessly, covering the following aspects:

The introduction sets the stage by providing a concise background and articulating the research's motivation. Following this, the modeling and mathematical formulation section introduces the theoretical underpinnings essential for understanding the studied phenomena.

The numerical procedure details the methodology applied to solve governing partial differential equations, employing the finite element method. Subsequently, the article presents a detailed analysis of the obtained results, offering insights into flow patterns, temperature distributions, and other pertinent parameters within the semicircular cavity.

The conclusion succinctly summarizes the major findings and their implications. Lastly, the references section provides a comprehensive list of all cited sources utilized in the article.

This cohesive structure guides readers through a logical progression of problem introduction, theoretical framework, numerical approach, analysis, and conclusion, enriching our understanding of natural convection complexities in intricate geometries involving MHD fluid.

CHAPTER 3

Methodology

3.1 COMSOL Multiphysics® Simulation Software:-

Engineers and scientists use the COMSOL Multiphysics® software to simulate designs, devices, and processes in all fields of engineering, manufacturing, and scientific research. COMSOL Multiphysics® is a simulation platform that provides fully coupled multiphysics and single-physics modeling capabilities. The Model Builder includes all of the steps in the modeling workflow — from defining geometries, material properties, and the physics that describe specific phenomena to performing computations and evaluating the results.

When we have developed a model, we can use the Application Builder to turn it into a simulation application with a dedicated user interface that can be used by collaborators and customers who are not experts in simulation software. To help keep our models and applications organized, the COMSOL Multiphysics® platform also includes the Model Manager, which is a tool for modeling and simulation management that provides version control and efficient storage.

You can add any combination of add-on products from the COMSOL product suite to the COMSOL Multiphysics® software platform. Doing so gives you access to specialized features to suit your particular modeling needs in a user interface that always looks the same, regardless of engineering field or physics phenomena.

3.2 TECPLOT VISUALIZATION & ANALYSIS

Tecplot is a software suite widely used in engineering and scientific fields for visualizing and analyzing simulation and experimental data. It provides powerful tools for creating 2D and 3D plots, visualizing complex datasets, and extracting meaningful insights from numerical simulations. Tecplot is commonly employed in disciplines such as fluid dynamics, aerospace engineering, oil and gas, and other scientific and engineering domains. Key features of Tecplot software include:
Data Import and Compatibility: Tecplot supports a wide range of data formats, allowing users to import data from various sources such as simulation codes, CAD software, and experimental measurements.

2D and 3D Plotting: Users can create high-quality 2D and 3D plots to visualize scalar and vector field data. This includes contour plots, surface plots, vector plots, streamline plots, and more.

Data Analysis Tools: Tecplot offers tools for analyzing and manipulating data, including data extraction, averaging, integration, and filtering. Users can perform quantitative analysis to extract important information from their datasets.

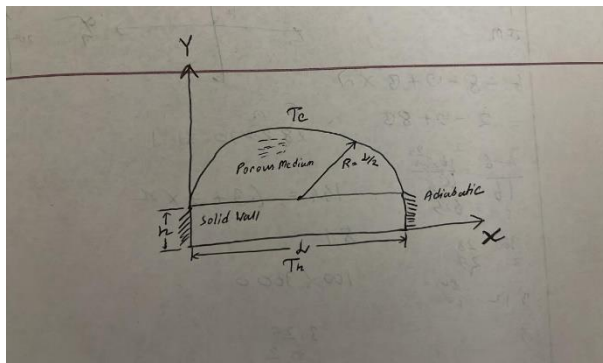
Automation and Scripting: Tecplot provides scripting capabilities, allowing users to automate repetitive tasks and create custom scripts for specific analyses. This can be particularly useful for batch processing and customization.

CFD and FEA Integration: Tecplot is often used in conjunction with Computational Fluid Dynamics (CFD) and Finite Element Analysis (FEA) software. It can read and visualize results from popular simulation tools.

Multi-Frame Workflows: Users can set up multi-frame layouts to compare different time steps or scenarios within a single visualization, enabling a comprehensive analysis of dynamic simulations.

Publication-Quality Output: Tecplot allows users to generate high-resolution, publication-quality images and animations for presentations and reports.

3.3 Description of physical problem:



Upper wall is kept at cold temp T_c , the bottom wall is kept at hot temp T_h . the side walls are insulated. the cavity is full with fluid saturated porous medium, the bottom wall is a solid wall.

3.4. Governing equations

For present model, we have considered a steady, MHD, viscous, incompressible flow in xy – coordinate system. Further we are considering that channel is filled with porous medium and motion of the fluid depending upon Darcy’s law, which produce for the drag exerted by the porous medium. The viscous dissipation effect is neglected and system of governing partial differential equation depends upon continuity equation under the law of conservation of mass, momentum and energy equations:

$$\frac{\partial U}{\partial X} + \frac{\partial v}{\partial y} = 0 \dots\dots\dots(1)$$

$$\rho \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial y} \right) = -\frac{\partial P}{\partial X} + \mu \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu}{K} U, \dots\dots\dots(2)$$

$$\rho \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial y} \right) = -\frac{\partial P}{\partial Y} + \mu \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\mu}{K} V - \sigma \beta_0^2 V + g \rho \beta (T^0 - T_c), \dots\dots\dots(3)$$

$$\left(U \frac{\partial T^0}{\partial X} + V \frac{\partial T^0}{\partial y} \right) = x \left(\frac{\partial^2 T^0}{\partial X^2} + \frac{\partial^2 T^0}{\partial y^2} \right) \dots\dots\dots(4)$$

CHAPTER 4

Validation

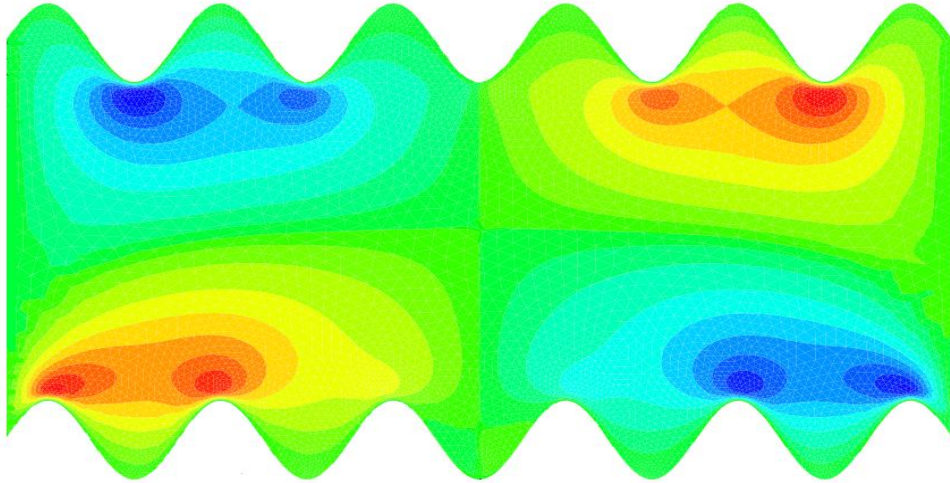


Fig:01

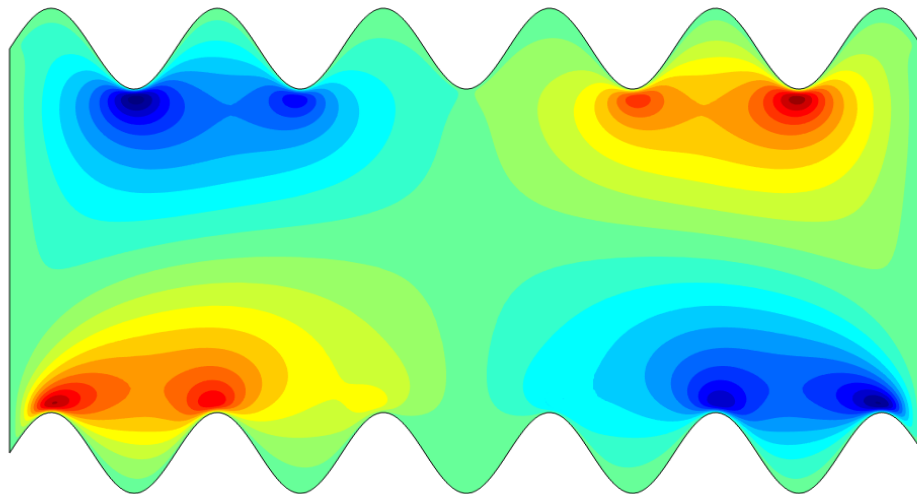


Fig:02

To verify the trustworthiness of any mathematical model and the associated simulation tool, any previously published studies have to be evaluated with the current simulation technique. Hence, the current methodology is validated using the results of Haq et al. [45]. From the figure, data suggests incredible agreement between the published result and the present methodology of work.

Velocity (u) had been determined for $Ra = 10^4$ when $Pr = 6.2$, $Ha = 10$, and $Da = 10^{-4}$

CHAPTER 5

Result and Discussion

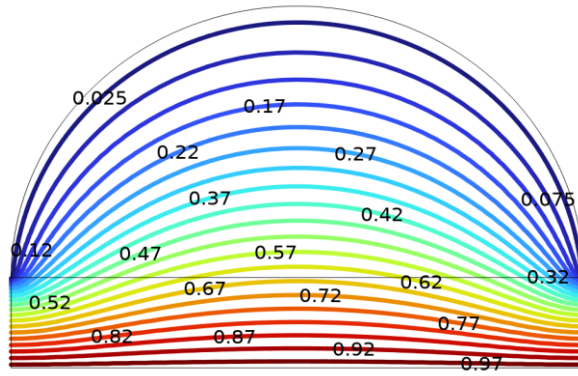


Figure:03

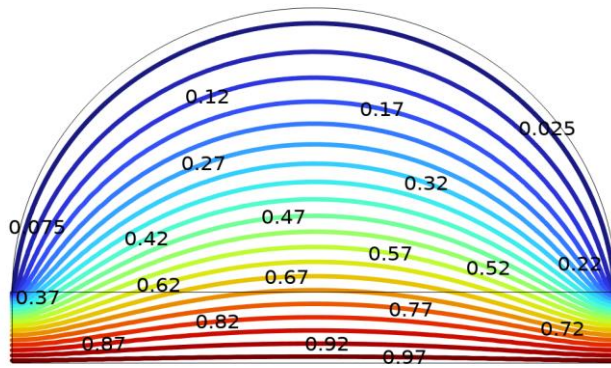


Figure:04

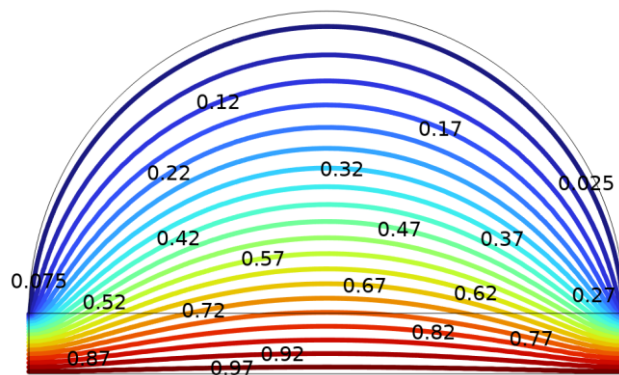


Figure:05

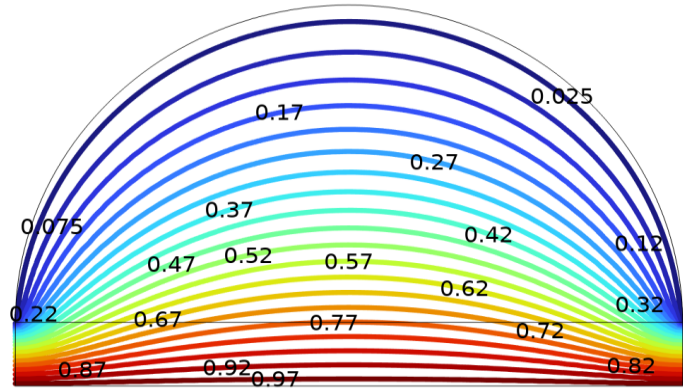


Figure:06

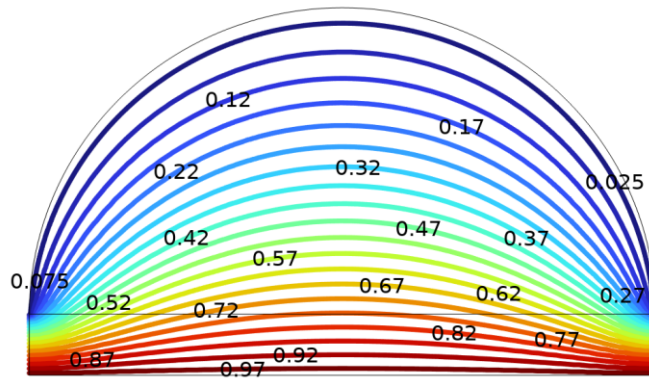


Figure:07

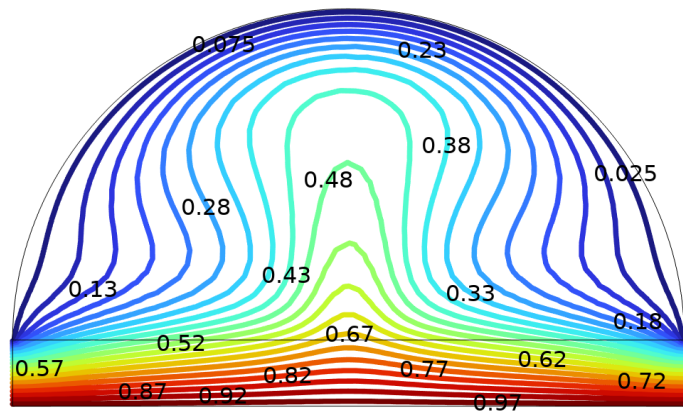


Figure:08

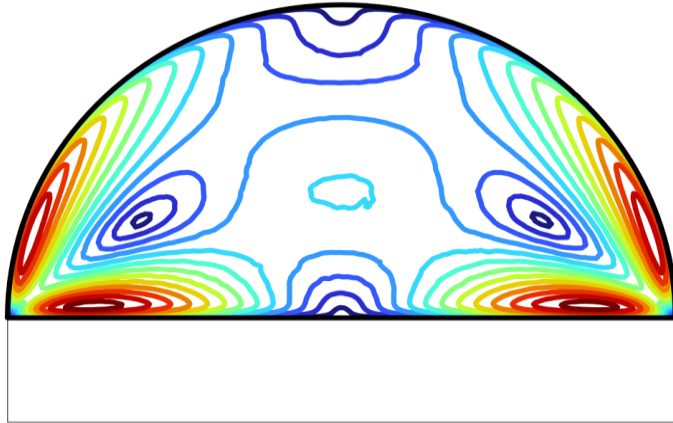


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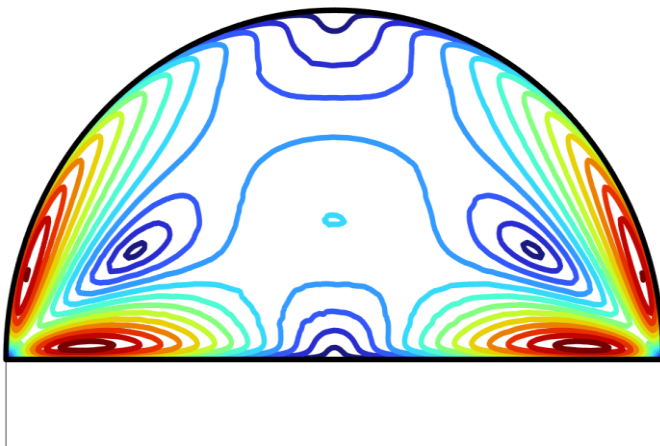


Figure:10

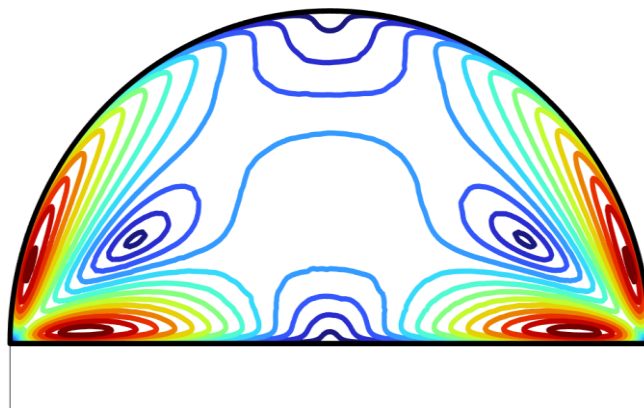


Figure:11

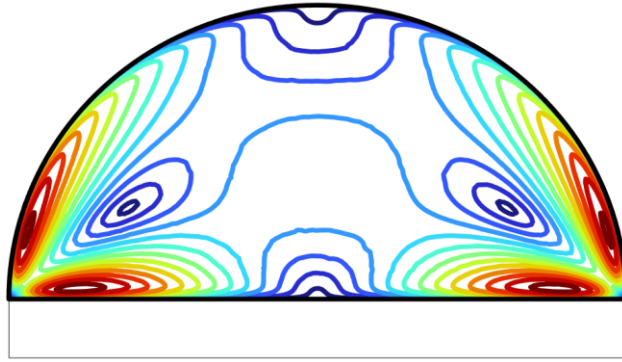


Figure:12

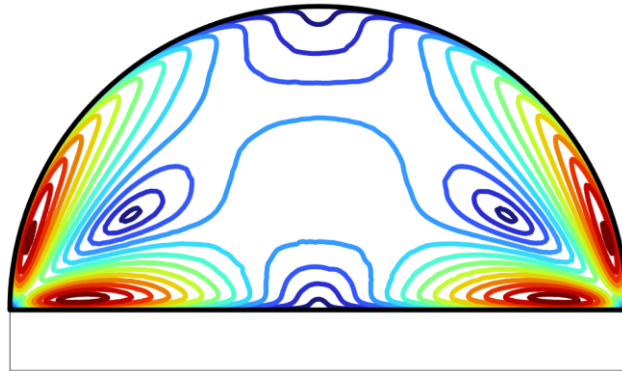


Figure:13

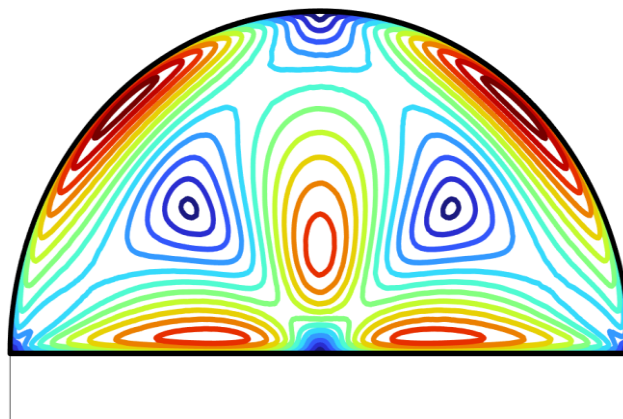


Figure:14

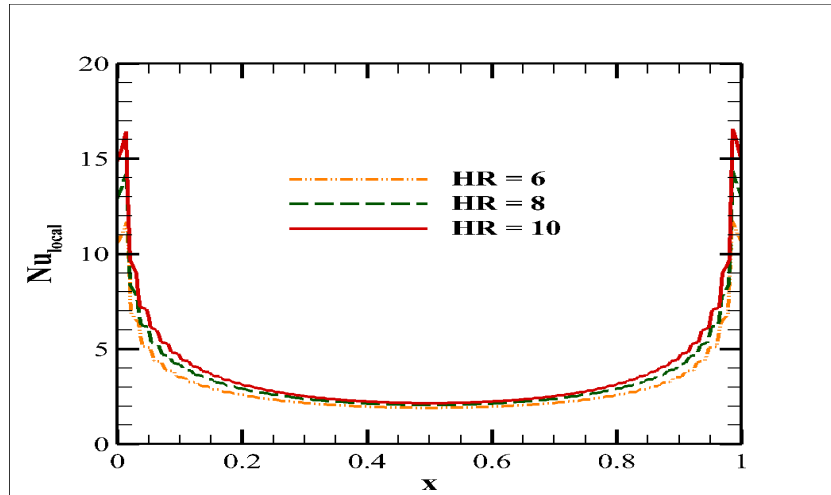


Figure:15

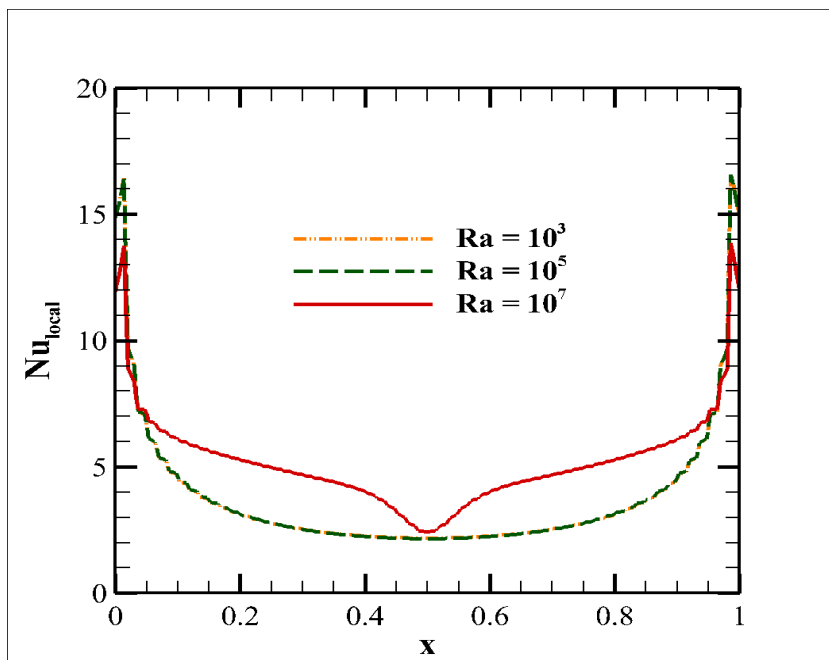


Figure:16

Rayleigh number: In a semicircular cavity, the heat transfer behavior is influenced by the Rayleigh number (Ra), which characterizes the intensity of natural convection within the fluid inside the cavity. The Rayleigh number is defined as the ratio of buoyancy forces to viscous forces and is given by:

$$Ra = \frac{g\beta\Delta TL^3}{\nu\alpha}$$

Here:

- g is the acceleration due to gravity,
- β is the thermal expansion coefficient,
- ΔT is the temperature difference,
- L is a characteristic length (related to the dimensions of the cavity),
- ν is the kinematic viscosity, and
- α is the thermal diffusivity.

When the Rayleigh number increases in a semicircular cavity, it indicates a higher ratio of buoyancy forces to viscous forces. This leads to more pronounced natural convection currents within the cavity. The fluid movement becomes more vigorous, enhancing the heat transfer process.

In the context of a semicircular cavity, the increased Rayleigh number typically corresponds to stronger and more complex flow patterns within the fluid. The intensified fluid motion allows for more effective heat transfer from the heated surfaces (such as the cavity walls) to the fluid.

In summary, as the Rayleigh number increases in a semicircular cavity, the associated enhancement in natural convection leads to increased fluid movement, resulting in improved heat transfer within the system.

CHAPTER 6

Conclusion and Future Recommendations

Conclusion:

In conclusion, the numerical investigation of natural convection in a semicircular cavity and subjected to a magnetic field has provided valuable insights into the complex interplay of fluid dynamics, heat transfer, and magnetic effects within such intricate geometries. The key findings and conclusions derived from this study can be summarized as follows:

Flow Patterns and Temperature Distributions:

The simulations have revealed intricate flow patterns and temperature distributions within the corrugated cavity, highlighting the impact of both natural convection and magnetic field interactions.

Magnetohydrodynamic Effects:

The inclusion of magnetohydrodynamic effects has showcased their influence on fluid flow and temperature distribution, providing a nuanced understanding of how magnetic fields alter convective heat transfer.

Sensitivity to Parameters:

Sensitivity analyses have identified key parameters affecting the simulation results, contributing to a more nuanced comprehension of the system's behavior.

Validation and Model Reliability:

The numerical model has been validated against available analytical solutions or experimental data, establishing its reliability for predicting natural convection phenomena in corrugated cavities with porous media and magnetic fields.

Future Recommendations:

While this study has provided valuable insights, several avenues for future research and improvement emerge:

Extended Parametric Studies:

Conduct further parametric studies, exploring a broader range of values for parameters such as wavelength, Rayleigh number, Darcy number, and Hartmann number. This will enhance the understanding of the system's response to varying conditions.

Advanced Numerical Techniques:

Investigate the application of advanced numerical techniques or alternative simulation approaches to further improve the accuracy and computational efficiency of the model.

Experimental Validation:

Consider experimental validation to corroborate the numerical findings and provide a more comprehensive assessment of the model's accuracy under real-world conditions.

Incorporation of Nanofluids:

Extend the study to include nanofluids within the porous medium, exploring how the addition of nanoparticles influences natural convection and heat transfer.

Real-World Applications:

Explore practical applications of the research findings, particularly in the optimization of energy usage, design of efficient thermal systems, and development of sustainable technologies.

Further Magnetohydrodynamic Studies:

Delve deeper into magnetohydrodynamic effects by considering varying magnetic field strengths and orientations, offering a more comprehensive understanding of their impact.

Multiscale Modeling:

Consider adopting multiscale modeling approaches to capture finer details at the microscopic level within the porous medium, providing a more accurate representation of real-world scenarios.

This thesis lays the groundwork for further exploration and refinement in the understanding of natural convection phenomena in complex geometries, paving the way for advancements in heat transfer research and applications.

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